MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2015/2016

PEM0036 – CALCULUS

(Foundation in Engineering – All sections / Groups)

8 MARCH 2016
9.00 a.m. – 11.00 a.m.
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This Question paper consists of 6 pages.

2. Attempt **ALL FOUR** questions. All questions carry equal marks and the distribution of the marks for each question is given.

3. Please write all your answers in the Answer Booklet provided. All necessary working **MUST** be shown.

4. Only **NON-PROGRAMMABLE** calculator is allowed.

APPENDIX

BASIC DIFFERENTIATION AND INTEGRATION FORMULAS

$$\frac{d}{dx}[\sin x]=\cos x \qquad \frac{d}{dx}[\cos x]=-\sin x$$

$$\frac{d}{dx}[\tan x]=\sec^2 x \qquad \frac{d}{dx}[\cot x]=-\csc^2 x$$

$$\frac{d}{dx}[\sec x]=\sec x\tan x \qquad \frac{d}{dx}[\csc x]=-\csc x\cot x$$

$$\frac{d}{dx}[e^x]=e^x \qquad \frac{d}{dx}[\ln x]=\frac{1}{x};\quad x>1$$

$$\frac{d}{dx}[\sin^{-1} x]=\frac{1}{\sqrt{1-x^2}};\quad |x|<1 \qquad \frac{d}{dx}[\cos^{-1} x]=-\frac{1}{\sqrt{1-x^2}};\quad |x|<1$$

$$\frac{d}{dx}[\tan^{-1} x]=\frac{1}{1+x^2} \qquad \frac{d}{dx}[\cot^{-1} x]=-\frac{1}{1+x^2}$$

$$\frac{d}{dx}[\sec^{-1} x]=\frac{1}{|x|\sqrt{x^2-1}};\quad |x|>1 \qquad \frac{d}{dx}[\csc^{-1} x]=-\frac{1}{|x|\sqrt{x^2-1}};\quad |x|>1$$

$$Area=\int_a^b [f(x)-g(x)]\,dx$$

$$Volume\ (Disk)=\pi\int_a^b [f(x)]^2\,dx$$

$$Volume\ (Washer)=\pi\int_a^b [f(x)]^2-[g(x)]^2\,dx$$

$$Volume\ (Cylindrical\ Shells)=\int_a^b 2\pi(shell\ radius)(shell\ height)\,dx$$

Continued...

QUESTION 1 [25 marks]

a. The function $f(x)$ is defined by

$$f(x)=\begin{cases} a\sqrt{x}+4b, & x\le 9 \\ x+6a-8b, & 9<x\le 11 \\ (x-5)^2, & x>11 \end{cases}$$

Determine values of the constants a and b for which $f(x)$ is continuous at every x.

[8 marks]

b. Evaluate the following limits.

i. $\lim\limits_{x\to 1} \dfrac{x-\sqrt[4]{x}}{x-1}$ [3 marks]

ii. $\lim\limits_{x\to -8} \dfrac{|x+8|}{x+8}$ [5 marks]

c. Find the vertical and horizontal asymptotes of $g(x)=\dfrac{3x-2}{x^3-2x^2-15x}$.

[9 marks]

Continued...

QUESTION 2 [25 marks]

a. Find equations of all lines in which their slope is -2 that are tangent to the curve $y = \dfrac{8}{x-2}$. [6 marks]

b. Assuming that the equation of x and y are defined implicitly as differentiable functions $x = f(t)$ and $y = g(t)$ respectively, determine the slope of the curve $t = 3$.

$2x^3 + 3t^2 - 28 = 0$ and $y^{-1} - 4t^2 - 92 = 0$

[4 marks]

c. Determine the absolute extreme values of the function $f(x) = -\dfrac{2}{e^{x^2}}$ if the interval is $(-\infty, +\infty)$. [5 marks]

d. For $g(x) = x \ln|2x|$, do the following.

i. Find the open intervals on which the function is increasing or decreasing. [6 marks]

ii. Use first derivative to identify the function's local extreme point(s). Next, verify the local extreme point(s) using second derivative. [4 marks]

Continued...

QUESTION 3 [25 marks]

a. Evaluate the following integrals.

i. $\int \frac{x^2+6x}{(x+3)^2}\,dx$ [5 marks]

ii. $\int x\left(\ln\sqrt{x}\right)\,dx$ [6 marks]

b. Show that $\int_0^{\sqrt{\pi/2}} x\sin^3\left(x^2\right)\,dx = \frac{1}{3}$. [5 marks]

c. Find the volumes of the solids generated by revolving the regions about the given axes. The region is bounded by $y = 4x^2$ and $y = 6x$ revolved about

i. the x-axis [4 marks]

ii. the y-axis [5 marks]

Continued...

QUESTION 4 [25 marks]

a. Verify whether $y = e^{-x}\tan^{-1}\left(4e^{x}\right)$ is the solution of the differential equation $y'+y=\dfrac{4}{1+16e^{2x}}$ with the initial condition $y(-\ln 4)=\pi$. [6 marks]

b. Solve the differential equation $\dfrac{dy}{dx}=5x^{4}\sec y$ if $y=\dfrac{\pi}{2}$ when $x=2$. Express the solution explicitly. Next, check whether $\left(3,\dfrac{\pi}{3}\right)$ satisfies the obtained solution.

[6 marks]

c. A certain differential equation is given as $\dfrac{d^{2}y}{dx^{2}}-3\dfrac{dy}{dx}=-7y$. Do the following.

i. Find the general solution of the differential equation. [7 marks]

ii. Find the particular solution of the differential equation if given the following:

$y(0)=1 \qquad y'(1)=0.$

$$\frac{dy}{dx}=\frac{1}{2}e^{3/2x}\left(3c_{1}+\sqrt{19}c_{2}\right)\cos\left(\frac{\sqrt{19}}{2}x\right)+\frac{1}{2}e^{3/2x}\left(3c_{2}-\sqrt{19}c_{1}\right)\sin\left(\frac{\sqrt{19}}{2}x\right)$$

You may write your answer in 3 decimals.

[6 marks]

End of Page